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Is mathematics similar to morality?

"The understanding of mathematics is necessary for a sound grasp of ethics"
~ Socrates

Introduction

Mathematics—the study of numbers, quantity and space—and morality—the study of what constitutes right and wrong—are two domains of knowledge that have fascinated philosophers for centuries. Under each domain of knowledge, we hold beliefs that I henceforth refer to as D-propositions. In determining whether mathematics and morality should be considered similar, I will consider three criteria that correspond to the Justified True Belief theory of knowledge:

1. How do we come to *believe* in D-propositions, and thus D-knowledge?
2. How *justified* is our belief in D-propositions?
3. What is the nature of the *truth* of D-propositions?

I will argue that mathematics and morality are dissimilar in terms of belief and justification, though they suffer the same ontological challenges with regard to truth.

How do we come to believe in D-propositions?

The process of forming mathematical and moral beliefs is highly distinct. Let us begin by considering the way in which we arrive at D-propositions, for example that '8 multiplied by 8 makes 64' or that 'killing dogs is wrong'. In the case of mathematical propositions, we start with a toolkit of axioms or mathematical rules, and then embark on a deductive chain in order to reach our conclusion. When multiplying 8 by 8, we start with the rule that $x \times y$ can be calculated by adding x to itself y times. From this rule, we then deduce that the product of 8 and itself is 64. Mathematical proofs, therefore, take the form of deduction-sketches.[1] On the other

[1] Justin Clarke-Doane, "Moral Epistemology: The Mathematics Analogy," *Noûs* 48, no. 2 (2014): 238–255.

hand, our moral beliefs usually parallel our intuitive emotional responses to certain actions. We then attempt to rationalise these responses through the application of moral theories or otherwise. My reaction towards a situation where someone has killed a dog is one of deep anger, translating into my intuition of the moral proposition that killing dogs is wrong. Following this, I might try to rationalise this normative belief, perhaps by appealing to basic principles such as ‘unnecessarily taking a life is immoral’. Hence, mathematical beliefs are formed from basic principles, followed by deduction, followed by a D-proposition, whereas moral beliefs instead start from the D-proposition, before looping back to basic principles to provide a post-hoc rationalisation of this belief.

This elucidates that both mathematical and moral beliefs are a priori: one’s reasons for thinking that a D-proposition is true is not derived from experience; thinking about it is enough to arrive at the belief. However, the two domains differ with regard to the *extent* of a prioricity. While both are independent of real-life and lived experience, moral beliefs—unlike mathematical beliefs—often involve some type of hypothetical or imagined experience. As Peacocke put it, moral theses have “strong prima facie support [...] from *consideration of examples*”.[2] This can be attributed to the nature of emotional intuition—it is exceedingly hard to feel an emotional response to abstract ideas that are too distant from us. (NB: I use the term ‘emotion’ loosely; it includes not only feelings like happiness/sadness but also an ingrained sense of righteousness or injustice.) To determine whether killing a dog is wrong, you would first have to consider the scenario in which a dog is killed to gauge your emotional intuition towards it, even if this consideration may be very brief. The tendency to rely on these hypothetical experiences can be seen in ethicists’ reliance on specific illustrations (e.g. in *On Liberty*, Mill establishes that hate speech is wrong through the example of telling “an excited mob assembled before the house of a corn dealer” that “corn dealers are starvers of the poor”, which intuitively has disastrous consequences).

On the contrary, forming mathematical beliefs rarely requires this sort of imagined experience, not least because it is hard to imagine what types of experience might

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[2] Christopher Peacocke, “Moral Rationalism,” *Journal of Philosophy* 101, no. 10 (2004): 499–526.

be relevant to mathematical propositions in the first place. How should one experience that the derivative of x^2 is $2x$? While moral beliefs involve imagined experience, mathematical beliefs seem divorced from any experience altogether.

This could also explain why there is a wider range of competing moral theories compared to mathematical ones. In the first place, moral propositions are more varied, because emotional intuition can differ widely from person to person. Whereas with mathematics, a broadly-accepted set of basic principles and relatively stringent rules of what constitutes valid deduction leads to a comparatively narrower range of D-propositions. Further, with moral beliefs, even if people hold the same intuitions, there are many possible ways to rationalise these action-level propositions—thus spawning a diversity of moral theories and, in turn, even more theory-level moral propositions. People in countries accustomed to consuming dog meat might simply not intuitively think that killing dogs is wrong. Even among the population that agree it is immoral, some may rationalise this through Kantian deontology (since everyone universally killing dogs is undesirable[3]) or utilitarianism (since killing a dog decreases the happiness of the dog and dog-lovers) or even virtue ethics (since we should be honourable and kind, which is incompatible with killing dogs). All of these modes of justification generate their own sets of propositions (in the Kantian case, that 'whatever we would not want to be a universal law of nature is immoral'). In line with the observations of descriptive moral relativism, moral beliefs thus seems to be a lot more diverse compared to mathematical beliefs.

How justified is our belief in D-propositions?

The foundationalist perspective posits that beliefs are justified if they are, or are deduced from, basic infallible beliefs. Under such an account, neither mathematical nor moral knowledge is mostly justified. There are infallible beliefs in both domains, in that propositions like '3+2=5' or 'it is right to act in a virtuous way' are self-evident and infallible. This is perhaps a matter of definition: '3+2' and '5' represent the same number because of how each symbol is defined[4], and the meaning of 'virtuous'

[3] If killing dogs became a universalised law, eventually, all dogs will be killed and we will be unable to continue killing dogs—giving rise to a contradiction. Hence, killing dogs cannot be a universal law.
[4] Carl Hempel, "On the Nature of Mathematical Truth," *American Mathematical Monthly* 52, no. 10 (1945): 543–556.

(which is 'having good moral qualities and behaviour') inherently implies moral praise. However, the set of basic beliefs hardly suffices as axioms from which most other beliefs can be deduced. In mathematics, such 'definitional' propositions cannot deductively lead to the propositions of calculus or geometry. Where there have been attempts to provide an axiomatic foundation (e.g. Zermelo-Fraenkel axioms) from which all, or at least a substantial part, of mathematical knowledge can be derived, axioms cannot be reliably proven to be infallible (consider the ZF axioms of replacement and infinity, which are not self-evident, or for any other reason I can think of, apparently infallible). In morality, action-level propositions would require some assertion of what *constitutes* virtue—independent from the fact that virtue is right—and theory-level propositions clearly cannot be deduced from simple tautologous beliefs.

Alternatively, we can turn to the coherentist theory, under which mathematics and morality are dissimilar, and belief in mathematical propositions is more justified than belief in moral propositions. For a proposition to be coherently justified, it must cohere with a set of beliefs, i.e. that there is mutual agreement, inferential connection and logical consistency among the propositions in a web of beliefs. Further, the "more and better the relations" between beliefs, the greater the extent of coherence and thus strength of justification.[5]

When considering the set of D-propositions in the context of the whole domain of knowledge, mathematical propositions seem more coherently justified. As established, the degree of variation among mathematical propositions is significantly smaller. Basic mathematical propositions—the four operations, the set of natural numbers—are almost entirely congruous with different branches of mathematics and different mathematical theorems. There are also inferential connections between various mathematical theorems, for example trigonometric identities being heavily used in the calculation of area in Cartesian geometry, or algebraic formulas being adopted in statistics. Mathematical knowledge, therefore, constitutes a system of interconnected and coherent propositions. (That being said, mathematical coherence is not absolute either, with some theories positing incompatible and thus inconsistent

[5] Geoffrey Sayre-McCord, "Coherentism and the Justification of Moral Beliefs," in *Ethical Theory*, ed. Russ Shafer-Landau (New Jersey: Blackwell Press, 2007), 123–139.

claims, such as Riemannian versus Euclidean geometry.) With morality, while the set of propositions set forth by specific moral theories may be internally consistent, it is easy to see how the propositions set forth by different moral theories could be contradictory (e.g. while utilitarianism supports sacrificing an individual for the greater good, Kantian deontology rejects this on the basis that humans should not be used as means to an end)—with morality, therefore, the set of beliefs that can coherently be held lacks the same breadth and comprehensiveness as mathematics. To the extent that a more comprehensive web of knowledge correlates to a greater strength of justification,[6] mathematical beliefs generally seem better justified than moral beliefs.

Notably, the mathematical disagreement alluded to earlier is exclusive to high-level academic study, whereas laymen who use everyday mathematical theories rarely find their mathematical propositions coming into conflict. In learning mathematics, individuals largely learn widely-accepted basic propositions and stringent deductive rules, entailing that if we consider the set of D-propositions that each individual holds, the set of mathematical propositions an individual believes in is also more likely to be coherent. An individual's moral beliefs tend to be incoherent because they are more likely to be inconsistent. An informal online quiz called the Philosophical Health Check presents users with a list of 30 metaethical and normative statements to agree or disagree with. The mean 'tension quotient'—the degree to which one's beliefs contradict one another—is 27%.[7] Obviously, the study is far from rigorous, but this does highlight how people's individual web of normative beliefs often include propositions that are incompatible with one another, which can be attributed to how our emotional intuitions are not fully consistent. For example, though many might intuitively agree that killing animals unnecessarily is wrong (by intuiting that it is cruel and unfair), they might still hold that there is nothing immoral with eating meat even though humans do not actually need to consume meat (since they have no negative emotional reaction to it). The moral beliefs held by individuals, therefore, are not as coherently justified as mathematical ones.

[6] Geoffrey Sayre-McCord, "Coherentism," 123-139.
[7] "Philosophical Health Check," Philosophy Experiments, accessed August 31, 2022, https://www.philosophyexperiments.com/health/Default.aspx.

Another distinct advantage that mathematics has is the fact that it coheres with a wider web of scientific knowledge, providing empirical and evidential support for mathematical propositions. This is true of the natural sciences in particular. The relationship between the propositions of natural sciences and mathematics goes a step beyond congruity, given that the laws of physics and chemistry like mass-energy equivalence or chemical stoichiometry are fundamentally dependent on the operations of mathematics, conferring this relationship an inferential quantity. Given further that these scientific laws evidentially cohere with empirical observations of phenomena in the real world, the coherentist justification for mathematical beliefs seems even stronger. On the flip side, a similar type of empirical support is difficult for moral propositions. Hume's is-ought gap reveals that normative propositions cannot be inferred or deduced from non-moral—including empirical—evidence. Hence, even if we accept that moral beliefs may be consistent with legal or social scientific knowledge, the strength of their relation is limited by the nature of that relation. Insofar as empirical support is significant in establishing the soundness of coherentist justification by enhancing the strength of neighbouring beliefs, mathematical beliefs seem more justified than moral beliefs.

What is the nature of the truth of D-propositions?

The area of truth we will focus on is ontological: namely, the debate between realism and anti-realism. The realist position claims that (1) the world exists objectively and mind-independently, and (2) our propositions purport facts about that world, and are true if they align with the facts.[8] Conversely, truth in the anti-realist view is either non-objective or outright impossible. (NB: Ultimately, whether mathematical and moral entities are ontologically real should not be mistaken as a direct proxy of the extent to which D-propositions are likely true; these are two separate issues.)

A key argument in support of mathematical realism is the Quine-Putnam Indispensability Argument: that mathematical hypotheses, unlike moral hypotheses, are indispensable to our best empirical scientific theories, allowing us at least defeasible empirical justification for believing that hypothesis. Some have tried to

[8] Michael Glanzberg, "Truth," The Stanford Encyclopedia of Philosophy, last modified June 21, 2022, https://plato.stanford.edu/archives/sum2021/entries/truth/.

argue that while moral hypotheses may not be relevant to scientific theories, they nonetheless play a parallel indispensable explanatory role in other kinds of empirical observations. Philosophers have proposed several scenarios to illustrate this, for instance that the growth of political protest movements can be explained by the injustice—a normative quality—of society.[9] However, an issue with this argument is that these sorts of normative justifications still necessarily "implicate human mental states".[10] There is a missing middle step in the explanation: holding the moral belief that 'society is unjust' results in feelings of frustration, which *in turn* explains the rise of political movements. The explanatory role of moral propositions stems from an individual's *belief* in the proposition, not the actual *accuracy* of the proposition—people holding a proposition that does not correspond with 'real' moral truths could very well still explain the relevant phenomena. Insofar as we consider the indispensability argument, we ought to be ontologically committed to mathematical propositions, but not necessarily moral ones.

However, a crucial parallel between mathematics and morality is the following: if we consider mind-independent mathematical or moral entities to ontologically exist, they are abstract objects that would not exist in our spatio-temporal world.[11] This challenges realism in both domains of knowledge in two ways: (1) unlike the empirical sciences where we can perceive physical objects through our senses, how can we confirm that the world of mathematical/moral facts exists? (2) even if we accept that it exists, how can we make propositions about objects in a world we have no idea about? The upshot of (2) is that we do not seem to be *discovering* mathematical/moral facts, as there is no causal relationship between perceiving the facts of mathematical/moral objects and our formation of mathematical/moral beliefs—it seems that our propositions cannot purport facts about mathematical/moral entities. More likely then, the ontological status of mathematical and moral objects are similar in that it seems, at best, indeterminate.

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9 Debbie Roberts, "Explanatory Indispensability Arguments in Metaethics and Philosophy of Mathematics," in *Explanation in Ethics and Mathematics*, ed. Uri Leibowitz and Neil Sinclair (Oxford: Oxford University Press, 2016), 185–203.
10 Michael Gill, "Morality is Not Like Mathematics: The Weakness of the Math-Moral Analogy," *Southern Journal of Philosophy* 57, no. 2 (2019): 194–216.
11 Adam Grief, "Parfit on Moral Disagreement and The Analogy Between Morality and Mathematics," *Filozofia* 76, no. 9 (2021): 688–703.

Conclusion

I have established that mathematics and morality are similar insofar as the realist view towards both is doubtful. However, I have shown that belief in D-propositions is derived in distinct ways, and that justification for mathematical propositions tends to be stronger than for moral propositions. Altogether, it seems that mathematics is largely distinct from morality.

Mathematics and morality are fundamentally two different subjects, with a long list of differences—I have skipped most of the trivial comparisons in order to draw meaningful analogies, but owing to the word count, there remain many more areas to explore.

At the end of the day, mathematical and moral propositions both have great pragmatic value. Mathematics has not just helped us in our everyday lives, its role in the practical sciences has also helped us better understand the physical phenomena we observe and experience rapid technological development. Moral beliefs, in influencing our intuitive emotional responses to certain actions, have helped us govern our own and others' behaviours so that society can function properly. As epistemological and ontological debates carry on, it is important not to devalue the robust practical benefits that both mathematical and moral study have given us.

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